Part One. Multiple choice (50%). The first 20 problems are multiple choice. Fill in the letter of the best answer on your bubble sheets. There is no penalty for a wrong answer. YOU MUST ALSO WRITE YOUR ANSWER AND SHOW ALL YOUR WORK IN YOUR BLUE BOOK(S).

1. The triangle enclosed by the straight lines $y = x$, $y = 2x$, $x = 2$ has area

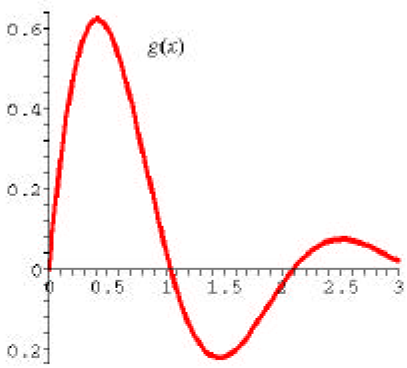
(a) 2 (b) 3 (c) 4 (d) 3.6 (e) 1.75

2. The substitution $u = \sqrt{x}$ transforms the integral $\int_1^4 \frac{f\left(\sqrt{x}\right)}{\sqrt{x}}dx$ into

(a) $\int_1^4 \frac{f(u)}{u}du$ (b) $2\int_1^4 f(u)\,du$ (c) $\int_1^2 uf(u)\,du$ (d) $\int_1^2 f(u)\,du$ (e) $2\int_1^2 f(u)\,du$

3. Given that $f(1) = 3$, $\int_0^1 xf'(x)\,dx = 5$, then $\int_0^1 f(x)\,dx$ equals

(a) 2 (b) –2 (c) 0 (d) –3 (e) 3



4. Given the graph of $y = g(x)$ on the left, if we define $G(x) = \int_0^x g(t)dt$, then G is ***increasing*** on the interval

(a) $0 < x < 0.5$ and $1.5 < x < 2.5$
(b) $0 < x < 1$ and $2 < x < 3$
(c) $0 < x < 0.5$ and $1.5 < x < 2$
(d) $1 < x < 2$
(e) $0.5 < x < 1.5$ and $2.5 < x < 3$

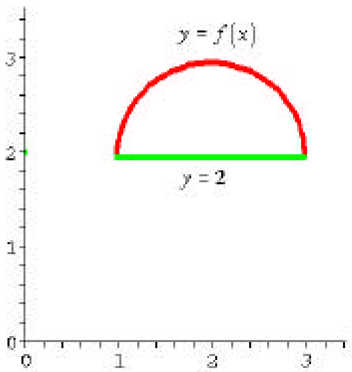
x	0	2	4	6	8
$f(x)$	9	25	16	4	9

5. Given the values of $f(x)$, then Simpson's rule S_4 for the integral $\int_0^8 \sqrt{f(x)}\,dx$ yields the approximation

(a) 28 (b) 42 (c) 332/3 (d) 36 (e) 86/3

6. If $\int_1^\infty f(x)\,dx$ converges, then $\int_1^t f(x)\,dx$ could be

(a) $e^t - 1$ (b) $\frac{t^2+1}{t}$ (c) $\frac{e^t}{t}$ (d) $\ln\left(\frac{1}{t}\right)$ (e) $\frac{\ln(t)}{t} + 1$



7. If the region bounded by $y = f(x)$ and $y = 2$ between $x = 1$ and $x = 3$ is rotated about the x-axis, the volume generated is given by

(a) $\int_1^3 (f(x) - 2)dx$ (b) $p\int_1^3 (f(x) - 2)^2 dx$

(c) $2p\int_1^3 x(f(x) - 2)dx$ (d) $p\int_1^3 ((f(x))^2 - 4)dx$

(e) $p\int_1^3 (2 - (f(x))^2)dx$

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8. Water has weight density of $9800 N/m^3$. The work needed to pump all the water in a full rectangular tank of base 3m by 4m and height 10m out the top of the tank is closest to
(a) $4.3x10^4$J (b) $8.2x10^4$J (c) $5.9x10^6$J (d) $4x10^6$J (e) $7x10^7$J

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9. A polar form of the Cartesian equation $(x-1)^2 + y^2 = 1$ is

(a) $r = 2\cos(q)$ (b) $r = 2\sin(q)$ (c) $r = \cos(q)$ (d) $r = \sin(q)$ (e) $r^2 = 2\cos(q)$

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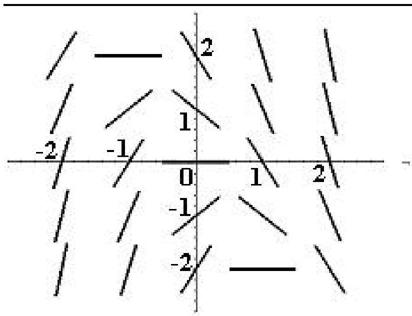
10. If $\cos(kt)$ is a solution of the differential equation $2\frac{d^2y}{dt^2} + 32y = 0$, then k could be

(a) 32 (b) 16 (c) 8 (d) 4 (e) 2



11. The direction field at left is for a differential equation of the form $y' = ax + by$. The values of a and b satisfy

(a) $a > 0, b > 0$ (b) $a > 0, b < 0$ (c) $a < 0, b > 0$

(d) $a = 0, b = 0$ (e) $a < 0, b < 0$

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12. For the direction field from problem #11, suppose $y(0) = 1$. Applying Euler's method with step size $h = 1$, would give the estimate $y(2) =$
(a) -2 (b) 2 (c) 0 (d) 1 (e) 3

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13. A simple electric circuit contains a 25-Ohm resistor and a 0.005-farad capacitor and an EMF of 125 volts. If $I(t)$ is the current and $Q(t)$ is the charge on the capacitor at time t(seconds), then Kirchhoff's law (after simplification) gives the differential equation

(a) $\frac{dI}{dt} + 8I = 5$ (b) $\frac{dI}{dt} + 0.002I = 5$ (c) $\frac{dQ}{dt} + 8Q = 5$ (d) $\frac{dQ}{dt} + 0.002Q = 5$

(e) None of these

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14. Consider a radioactive substance undergoing exponential decay. If 80% of the substance remains after 1000 years, then its half-life in years is closest to
(a) 3302 (b) 2182 (c) 3802 (d) 2254 (e) 3106

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15. The sum of the areas of an infinite sequence of circles of radii 1, 1/2, 1/4, 1/8, … is
(a) 2π (b) $3\pi/2$ (c) $4\pi/3$ (d) $5\pi/4$ (e) ∞

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16. The coefficient a_3 in the Taylor series $\sin(6x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ is
(a) 6^3 (b) -6^3 (c) 0 (d) 6^2 (e) -6^2

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17. The radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{223n}{9^n}(x-7)^n$ is
(a) 1/9 (b) 7 (c) 9 (d) 2007 (e) ∞

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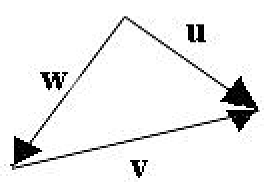
18. A function has a minimum at the point (1,2). Which of the following could be the Taylor series for $f(x)$ about $x=1$?
(a) $2+(x-1)^2+\cdots$ (b) $1-\frac{1}{2!}(x-2)^2+\cdots$ (c) $2+(x-1)-\frac{1}{2!}(x-1)^2+\cdots$
(d) $1+(x-1)+\frac{1}{2!}(x-1)^2+\cdots$ (e) $2-\frac{1}{2!}(x-1)^2+\cdots$



19. A a correct relationship among the vectors **u**, **v**, and **w**, is
(a) **v=u+w** (b) **u=v+w** (c) **w=u+v**
(d) **u=w-v** (e) **v=w-u**

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20. The scalar projection $comp_{\mathbf{b}}\mathbf{a}$ of vector $\mathbf{a} = \langle 2,-1,5\rangle$ in the direction of vector $\mathbf{b} = \langle 3,6,2\rangle$ is
(a) $\frac{10}{7}$ (b) $-\frac{10}{7}$ (c) $\left\langle 0,0,-\frac{10}{7}\right\rangle$ (d) $\langle 2,-1,-5\rangle$ (e) none of the previous

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Part Two. Longer Answers(50%). SOLVE 10 OF THE REMAINING 11 PROBLEMS. YOUR INSTRUCTOR MAY TELL YOU WHICH PROBLEMS TO SOLVE. OTHERWISE CHOOSE 10 OF THE PROBLEMS. These are **not** multiple-choice problems. Show all work and put your answers in your blue book(s).

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21. (a) State clearly both parts of the Fundamental Theorem of Calculus, including all assumptions.
(b) Use the Fundamental Theorem of Calculus to evaluate each of the following derivatives.

(i) $\dfrac{d}{dx}\int_{10}^{x} e^{t^2}\,dt$ (ii) $\dfrac{d}{dx}\int_{x}^{-10} e^{t^2}\,dt$ (iii) $\dfrac{d}{dx}\int_{1}^{x^3} e^{t^2}\,dt$

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22. Evaluate each of the following integrals without using your calculator. Show all work for credit. You may use your calculator in the end to check your answers.

(a) $\int \dfrac{x}{4x^2+1}\,dx$ (b) $\int t^3 \ln(t)\,dt$ (c) $\int y\sin(y)\,dy$

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23. (a) Write each improper integral below as a limit. (b) Compute this limit to determine whether the integral is convergent or divergent. If the integral is convergent, evaluate it.

(a) $\int_0^{\infty} te^{-t^2}\,dt$ (b) $\int_1^2 \dfrac{1}{(x-1)^2}\,dx$ (c) $\int_0^{\sqrt{p/2}} q\sec^2\left(q^2\right)dq$

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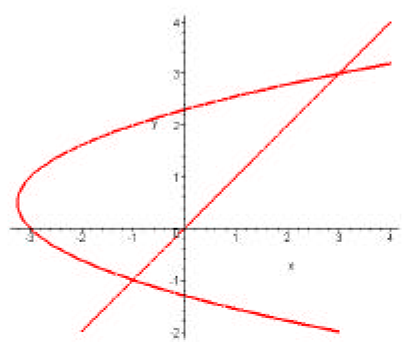
24. Use integration to find the exact area of the region bounded by $y = x$ and $x = y^2 - y - 3$.



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25. A swimming pool is 20 meters long. The vertical cross-sectional area x meters from one end is $A(x)$ square meters. Use the approximate integration rule of your choice to estimate the volume of a swimming pool, if the following is known.

x	0	4	8	12	16	20
$A(x)$	80	120	150	180	140	100

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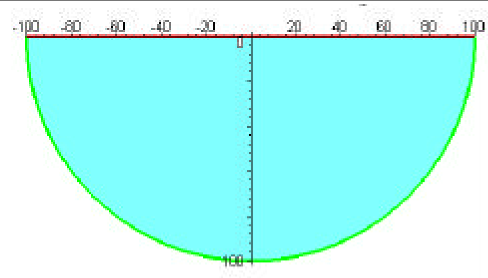
26. A dam has the shape of a semicircle of radius 100 feet. If the water reaches the top of the dam, find the total force due to hydrostatic pressure on the dam. Use the fact that the weight density of water is 62.5 lb/ft^3.



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27. (a) Sketch on the same graph the graphs of the polar curves: $r = 2\sin(q)$ and $r = 1$.
(b) Find polar coordinates for the intersection points of these curves.
(c) Find the area of the region inside the graph of the first curve and outside the graph of the second one.

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28. Consider the initial value problem $\frac{dy}{dx} - \frac{2}{x} y = x^2 \cos(x), \quad y(p) = p^2$.

(a) Find an integrating factor and use it to solve the differential equation.

(b) Solve the initial value problem and find $y\left(\frac{p}{2}\right)$.

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29. A simple series electric circuit consists of a 5-Henry inductor, a 10-Ohm resistor, and a 20-Volt battery. The initial current is 0. Let $I(t)$ be the current after t seconds.

(a) Set up an initial value problem with unknown function $I(t)$.

(b) Solve the differential equation **as separable** and then compute $I(t)$.

(c) Find the steady state current $\lim_{t\to\infty} I(t)$.

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30. (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n+10} n^2}{4^n}$ converges or diverges. Give your reasoning and show your work.

(b) Given that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $|x| < 1$, find the Maclaurin series for $\frac{1}{1+x^2}$.

(c) Use integration and the first three nonzero terms of the answer to part (b) to estimate $\arctan(0.2)$.

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31. Consider the vectors $\mathbf{a} = \overrightarrow{PQ}$ and $\mathbf{b} = \overrightarrow{PR}$, where P, Q, R are the space points $P(2,0,-3)$, $Q(3,1,0)$ and $R(5,2,2)$.

(a) Find the dot product $\mathbf{a} \bullet \mathbf{b}$.
(b) Find the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ to the nearest degree.
(c) Find the vector projection $proj_{\mathbf{b}}(\mathbf{a})$ of vector $\mathbf{a}$ in the direction of vector $\mathbf{b}$.
(d) Use a dot product to show that vector $\mathbf{a}$ is orthogonal to the cross product $\mathbf{a} \times \mathbf{b}$.
(e) Use the cross product $\mathbf{a} \times \mathbf{b}$ to find the area of the triangle PQR.